FRESHMAN PLACEMENT EXAMINATION 1967

CARD I

1. The value of $a^2 + 2ab + b^2$, when $a = -5$ and $b = 3$, is:

   a) -32 b) 19 c) -46 d) 4 e) -17

2. a^m divided by a^n =

   a) $a^{m/n}$ b) a^{m-n} c) $(m-n)\log a$ d) $\frac{m}{n} \log a$ e) 1

3. The product of $(5)^2 \cdot (-3)^0 \cdot (-4) \cdot (-2)^3$ =

   a) 800 b) -2400 c) 240 d) 120^6 e) -120^5

4. $(-3a^3)^3$ =

   a) $-3a^6$ b) $3a^9$ c) $-27a^9$ c) $27a^6$ e) $9a^3$

5. Express .0825 as a percent:

   a) 8.25% b) 82 1/2% c) 825% d) .82 1/2% e) .08 1/4%

6. If the dimensions of a rectangle are 3a and 4b, the area is:

   a) 12ab b) 3a + 4b c) $\frac{3a}{4b}$ d) 7ab e) $\frac{a^4b^3}{7}$

7. Simplify:

$$\frac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{a^2} - \frac{1}{b^2}} =$$

   a) $\frac{1}{a} - \frac{1}{b}$ b) $\frac{ab}{b - a}$ c) a - b d) $\frac{a - b}{ab}$ e) 1

8. If $5y^2$ is the quotient and $2xy$ the divisor, the dividend is:

   a) $\frac{5y}{2x}$ b) $y^2 - \frac{2x}{5y}$ c) $5y + 2x$ d) $10xy^3$ e) none of these

9. Simplify $\frac{6a + 10b}{2ab}$:

a) $3 + 5$ b) $\frac{3}{b} + \frac{5}{a}$ c) $12a^2b + 20\ ab^2$ d) $\frac{2ab}{6a + 10b}$

e) none of these

10. Dividing $x^3 - y^3$ by $x - y$ gives

a) $x^2 + y^2$ b) $(x+y)^2$ c) $x^2 + xy + y^2$ d) $(3x-3y)$ e) $(x-y)^2$

11. If $3y$ is an even integer, the next larger consecutive even integer is:

a) $4y$ b) $6y$ c) $3y + 2$ d) $3(y+1)$ e) $y + 3$

12. How many cubic yards of concrete are needed to build a sidewalk x feet long and y feet wide and 4 inches deep?

a) $4xy$ b) $\frac{4xy}{27}$ c) $36xy$ d) $\frac{4}{9}xy$ e) $\frac{1}{81}xy$

13. $3n$ is what percent of $12n$?

a) 4 b) $\frac{1}{4}$ c) 400 d) $9n$ e) 25

14. In $3x^n$, n is called the

a) exponent b) quotient c) logarithm d) dividend e) square

15. If the radius of a circle is doubled, the area is multiplied by:

a) 1 b) $\frac{3}{2}$ c) $\sqrt{2}$ d) 4 e) 2

16. $(a^2+2)^2 =$

a) $a^2 + 4$ b) $a^4 + 4$ c) $a^4 + 4a^2 + 4$ d) $a^4 + 4a + 2$

e) $a^2 + 2a + 4$

17. $\frac{6x + 6}{2x + 2} =$

a) $3 + 3$ b) 3 c) $4x + 4$ d) $3x + \frac{3}{x}$ e) $x + 3$

18. If $3a + b = 5$, $b =$

a) $\frac{5}{3a}$ b) $\frac{3a}{5}$ c) $2 - a$ d) $\frac{5 - a}{3}$ e) $5 - 3a$

19. If $a = -3$, the value of $-2(-3a)^2 =$

a) -162 b) -72 c) -18^2 d) 12^2 e) 18^2

20. Solve for x: $\frac{4x}{5} = \frac{2x + 1}{3} - \frac{4}{15}$

a) $-\frac{3}{2}$ b) $\frac{1}{2}$ c) $\frac{4}{3}$ d) 3 e) none of these

<u>CARD II</u>

21. A house sold for $13,200, which was 25% more than the original cost. The cost was:

a) 11,000 b) 13,000 c) 15,840 d) 9,900 e) 10,560

22. Factor completely: $x^2 - y^2 + 2x + 1$

a) $(x-y)(x+y)(2x+1)$ b) $x(x+2) + (1+y)(1-y)$ c) $(x+1)^2 - y^2$

d) $(x-y)^2(2x+1)$ e) $(x+y+1)(x-y+1)$

23. If the perimeter of a square is p. The area is

a) $(\frac{p}{4})^2$ b) $\frac{p^2}{4}$ c) $\pi\frac{p^2}{4}$ d) p^2 e) πp^2

24. $\frac{x^2+2y}{x^2y+xy^2} - \frac{x-y}{xy} =$

a) $\frac{x^2 - x + y}{x^2y + xy^2 - xy}$ b) $\frac{2}{y + xy} - 1$ c) $\frac{y + 2}{x^2 + xy}$

d) $2y + y^2$ e) $\frac{2y + y^2}{x + y}$

25. $\frac{x - y}{xy} =$

a) 0 b) 1 c) $\frac{1}{y} - \frac{1}{x}$ d) $\frac{xy}{x - y}$ e) none of these

26. $\dfrac{8}{9x^2 - 9x} - \dfrac{5}{6x^2 + 6x} = ?$

a) $\dfrac{3}{3x^2 - 15x}$ b) $\dfrac{13}{3x^2 - 3x}$ c) $\dfrac{3}{15x^2 - 3x}$

d) $\dfrac{3x^2 - 3x}{18x^3 - x}$ e) $\dfrac{x + 31}{18(x^3 - x)}$

27. How many revolutions will a bicycle wheel d feet in diameter make in travelling x feet?

a) $\dfrac{x}{d}$ b) $\dfrac{x}{\pi d}$ c) $\dfrac{\pi x}{d}$ d) $\dfrac{x}{2\pi d}$ e) $\dfrac{2\pi x}{d}$

28. Solve for x: $x^2 - 5x = +6$.

a) 1, -5 b) 5, -6 c) 6, -1 d) $\sqrt{5x + 6}$ e) $\dfrac{+6 + 5x}{x}$

Solve the following inequalities:

29. $7x + 3 > 5x + 6$

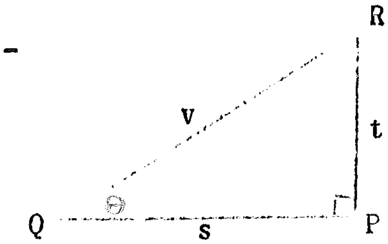
a) $x < \frac{3}{2}$ b) $x > 5$ c) $x < -1$ d) $x > \frac{3}{2}$ e) $x < -5$

30. $\dfrac{3x + 4}{2} < \dfrac{5x - 6}{2}$

a) $x > \frac{3}{2}$ b) $x < -5$ c) $x < -1$ d) $x < \frac{3}{2}$ e) $x > 5$

31. $\dfrac{2 - 3x}{5} > \dfrac{6 - x}{7}$

a) $x > \frac{3}{2}$ b) $x > 5$ c) $x < -5$ d) $x < \frac{3}{2}$ e) $x < -1$



Given the reference triangle QPR, with the angle PQR denoted by θ, and the angle QPR being a right angle.

32. $\sin\theta =$

a) $\frac{s}{t}$ b) $\frac{v}{s}$ c) $\frac{t}{v}$ d) $\frac{v}{t}$ e) $\frac{s}{v}$

33. $\sec\theta =$

a) $\frac{s}{t}$ b) $\frac{v}{s}$ c) $\frac{t}{v}$ d) $\frac{v}{t}$ e) $\frac{s}{v}$

34. $\cot\theta =$

a) $\frac{s}{t}$ b) $\frac{v}{s}$ c) $\frac{t}{v}$ d) $\frac{v}{t}$ e) $\frac{s}{v}$

35. $\text{arc}\cos\frac{1}{2} =$ (also written $\cos^{-1}(\frac{1}{2})$) = ?

a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{4}$ e) 0

36. The function which may be expressed $\tan[2\tan^{-1}x] =$ (or $\tan[2 \text{ arc} \tan x]$) = ?

a) $\frac{2x}{x^2 - 1}$ b) $\frac{x^2 - 1}{2x}$ c) $\frac{2x}{x^2 + 1}$ d) $\frac{x^2 + 1}{2x}$ e) $\frac{2x}{1 - x^2}$

37. Which of the following is <u>not</u> a "fundamental identity"?

a) $\sin^2\theta + \cos^2\theta = 1$ b) $\frac{\sin\theta}{\cos\theta} = \tan\theta$ c) $\sec\theta = \frac{1}{\cos\theta}$

d) $\cot\theta = \frac{\csc\theta}{\sec\theta}$ e) $\cot^2\theta + 1 = \csc^2\theta$

Solve the following two equations for values of the variable which are positive and not greater than 180°.

38. $2\sin^2 x - 3\sin x = -1$, $x =$

a) $\frac{\pi}{3}, \frac{\pi}{2}$ b) $\frac{\pi}{3}, 0$ c) $\frac{\pi}{6}, 0$ d) $\frac{\pi}{6}, \frac{\pi}{2}$ e) none of these

39. If $2\sin^2\theta = \cos\theta + 2$, $\theta =$

a) $\frac{\pi}{2}, \frac{2\pi}{3}$ b) $0, \frac{\pi}{3}$ c) $0, \frac{2\pi}{3}$ d) $\frac{\pi}{2}, \frac{\pi}{3}$ e) none of these

40. Evaluate without using tables:

$$\sin(\text{arc tan } \tfrac{1}{2} + \text{arc tan } \tfrac{1}{3})$$

a) $\frac{1}{\sqrt{2}}$ b) $\frac{3}{\sqrt{5}}$ c) $\frac{2}{\sqrt{10}}$ d) $\frac{13}{\sqrt{50}}$ e) $\frac{2}{3}$

FRESHMAN PLACEMENT EXAMINATION

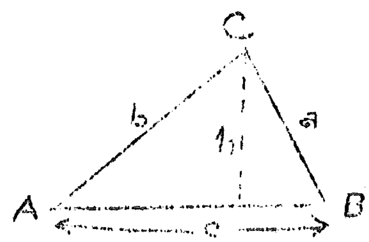
CARD III

1. $\log \frac{a}{b} = ?$

a) $\log a - \log b$ b) $\frac{\log a}{\log b}$ c) $\frac{a}{b}$ d) $\frac{\log a}{b}$

e) $\sqrt[b]{a}$

2. The "law of sines" or "sine law" is:

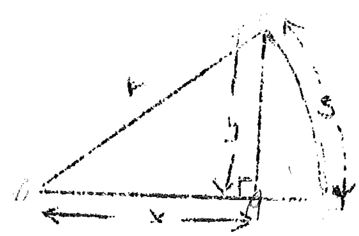


a) $\sin A = \frac{a}{b}$ b) $\sin A = \frac{h}{b}$

c) $\frac{\sin A}{a} = \frac{\sin B}{b}$ d) $\sin A = \frac{h}{a}$

e) $\frac{\sin A}{b} = \frac{\sin B}{a}$

3. In the figure at the right there is an angle θ, a right triangle OAB, and a segment of a circle OCB. The radian measure of the angle θ may be found by dividing:

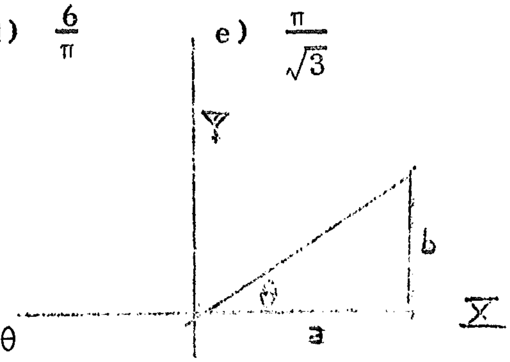


a) y by r b) s by x c) r by s d) x by y e) s by r

4. An angle of 30° has a radian measure of

a) $\frac{\pi}{6}$ b) $\sqrt{3}$ c) $\frac{1}{2}$ d) $\frac{6}{\pi}$ e) $\frac{\pi}{\sqrt{3}}$

5. The complex number a + ib can be expressed in trigonometric form as:



a) $\cos\theta + i \sin\theta$ b) $\sin\theta + i \cos\theta$

c) $r(\cos\theta + i \sin\theta)$ d) $r(\sin\theta + i \cos\theta)$

e) $\cos^2\theta + \sin^2\theta = 1$

6. The three solutions of the equation $x^3 = 8$ are $x = 2$ and the two complex numbers:

a) $-\sqrt{3} \pm i$ b) $-2 \pm 2i\sqrt{3}$ c) $-1 \pm i\sqrt{3}$

d) $-2\sqrt{3} \pm 2i$ e) none of these

7. $\sin(\alpha+\beta) =$

a) $\sin\alpha \cos\beta + \cos\alpha \sin\beta$ b) $\sin\alpha \cos\beta - \cos\alpha \sin\beta$

c) $\cos\alpha \cos\beta + \sin\alpha \sin\beta$ d) $\sin\alpha \sin\beta - \cos\alpha \cos\beta$

e) $\cos\alpha \cos\beta - \sin\alpha \sin\beta$

8. $\cos(\alpha+\beta) =$

a) $\sin\alpha \cos\beta + \cos\alpha \sin\beta$ b) $\sin\alpha \cos\beta - \cos\alpha \sin\beta$

c) $\cos\alpha \cos\beta + \sin\alpha \sin\beta$ d) $\sin\alpha \sin\beta - \cos\alpha \cos\beta$

e) $\cos\alpha \cos\beta - \sin\alpha \sin\beta$

9. $(i^6 + i^9 + i^{12})^5 =$

a) 0 b) i c) 1 d) $-i$ e) -1

10. $(\cos\alpha + i \sin\alpha)^2 =$

a) $\cos 2\alpha + i \sin 2\alpha$ b) $\sin\alpha \cos\alpha - \cos\alpha \sin\alpha$

c) $\cos 2\alpha - i \sin 2\alpha$ d) $\cos^2\alpha - \sin^2\alpha$

e) $\tan^2\alpha$

11. The equation of the circle with radius 1 and center at $(-\frac{3}{2}, \frac{1}{2})$ is

a) $x^2 + y^2 + 3x + y = 1$ b) $2x^2 + 2y^2 + 6x - 2y + 3 = 0$

c) $x^2 + y^2 - 3x - y = 1$ d) $2x^2 + 2y^2 - 6x + 2y + 3 = 0$

e) none of these

12. The slope of the line $\frac{x}{2} - \frac{y}{3} = 1$ is:

a) 1 b) $\frac{2}{3}$ c) $-\frac{3}{2}$ d) $-\frac{2}{3}$ e) $\frac{3}{2}$

13. The distance from (2,2) to the midpoint of the segment joining (2,3) with (-4,-1) is:

a) $2\sqrt{2}$ b) $\sqrt{3}$ c) $3\sqrt{2}$ d) $\sqrt{10}$ e) $\sqrt{13}$

14. The value of k that makes the lines $\begin{cases} 6x - 9y = 5 \\ kx - 4y = 8 \end{cases}$ perpendicular is:

a) -6 b) $-\frac{8}{3}$ c) $\frac{3}{8}$ d) $\frac{8}{3}$ e) 6

15. The parabola whose directrix is the line $y = -1$ and whose focus is (-1,3) is:

a) $x^2 + 2x = 8y$ b) $x^2 + 4x - y = -6$ c) $x^2 = 8y$

d) $2x^2 + 5y = 17$ e) $x^2 + 2x - 8y + 9 = 0$

16. The shortest distance between the circles $x^2 - 6x + y^2 + 5 = 0$, and $x^2 - 8y + y^2 + 15 = 0$ is

a) 0 b) 2 c) 1 d) $\sqrt{5}$ e) 5

In questions 17 and 18 assume the equation of the curve to be

$$4x^2 + 9y^2 + 24x - 18y = 36$$

17. The curve represented is a

a) circle b) parabola c) hyperbola d) ellipse

e) higher plane curve

18. The curve has its center at

a) (12,-9) b) (-12,9) c) (3,-1) d) (-3,1)

e) (6,-2)

In questions 19 and 20, assume the equation of the curve to be

$$4x^2 - 9y^2 + 36 = 0$$

19. The curve represented is a

a) circle b) hyperbola c) parabola d) ellipse

e) higher plane curve

20. As x increases without limit, y becomes

a) zero b) negative c) infinite d) ± 4 e) ± 1

Bob Marks

156

79

147-2) $7X^2-4XY+4Y^2=240$
A 7 B -4 C 4

$\cot 2\theta = \frac{7-4}{-4} = -\frac{3}{4}$ $\cos 2\theta = -\frac{3}{5}$

$\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}}$ $\cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}}$

$\sin\theta = \sqrt{\frac{4}{5}}$ $\cos\theta = \sqrt{\frac{1}{5}}$

$X = \frac{1}{\sqrt{5}}X' - \frac{2}{\sqrt{5}}Y' = \frac{X'-2Y'}{\sqrt{5}}$

$Y = \frac{2}{\sqrt{5}}X' + \frac{1}{\sqrt{5}}Y' = \frac{2X'+Y'}{\sqrt{5}}$

$\frac{7}{5}(X'-2Y')^2 - \frac{4}{5}(X'-2Y')(2X'+Y') + \frac{4}{5}(2X'+Y')^2 = 240$

$7(X'^2-4X'Y'+4Y'^2) - 4(2X'^2-3X'Y'-2Y'^2) + 4(4X'^2+4X'Y'+Y'^2) = 1200$

$15X'^2 + 40Y'^2 = 1200$

$3X'^2 + 8Y'^2 = 240$

$\frac{X'^2}{80} + \frac{Y'^2}{30} = 1$ ✓

X'Y' coordinates of vertices $(\pm 4\sqrt{5}, 0)$
XY " " " $(4,8)\ (-4,-8)$ ✓

147-4) $7X^2-6XY-Y^2=0$ A 7 B -6 C -1
$\cot 2\theta = -4/3$ $\cos 2\theta = -4/5$
$\sin\theta = \sqrt{9/10}$ $\cos\theta = \sqrt{1/10}$

$X = \frac{X'}{\sqrt{10}} - \frac{3Y'}{\sqrt{10}}$ $Y = \frac{3X'}{\sqrt{10}} + \frac{Y'}{\sqrt{10}}$

$X = \frac{X'-3Y'}{\sqrt{10}}$ $Y = \frac{3X'+Y'}{\sqrt{10}}$

$70(X'^2-6X'Y'+9Y'^2) - 60(3X'^2-8X'Y'-3Y'^2) +$
$-10(9X'^2+6X'Y'+Y'^2) = 0$

$70X'^2-420X'Y'+630Y'^2-180X'^2+480X'Y'+180Y'^2-90X'^2-60X'Y'-10Y'^2 = 0$

$-200X'^2+800Y'^2=0$

$4Y'^2 = X'^2$

$2Y' = \pm X' \Rightarrow$ 2 straight lines no vertices

in x-y ?

Bob Marks

174 - 23) PROVE $\log_b X = \dfrac{\log_a X}{\log_a b}$

Let $N = \log_b X \Rightarrow b^N = X$

$\log_a b^N = \log_a X$

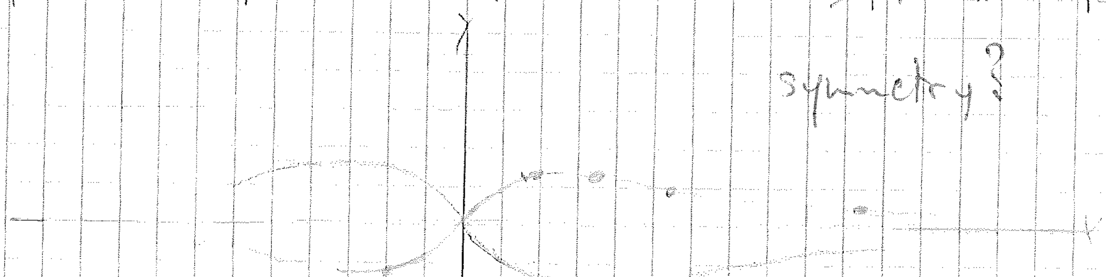
$(N \log_a b = \log_a X)$

$\therefore N = \dfrac{\log_a X}{\log_a b}$

OR $\log_b X = \dfrac{\log_a X}{\log_a b}$

187 - 26) $r = \cot \theta$

θ	0	$\pi/6$	$\pi/3$	$\pi/2$	$\pi/4$	$\frac{3}{4}\pi$	$\frac{2}{3}\pi$	$\frac{5}{6}\pi$	$\pi/12$	$\pi/18$
$\theta°$	0	30°	60°	90°	45°	135°	120°	150°	15°	10°
$\cot\theta$	—	$\sqrt{3}$	$\sqrt{3}/3$	0	—	-1	$-\sqrt{3}/3$	$-\sqrt{3}$	3.7	5.6
r	—	1.732	.58	0	—	-1	-.58	-1.734	3.7	5.7

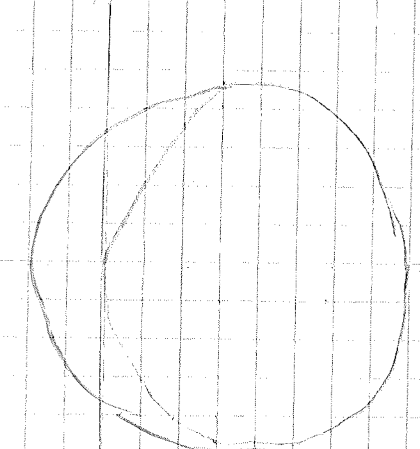


symmetry?

187 - 28) $r(2 - \cos\theta) = 4$

symmetry to X axis

$r = \dfrac{4}{2 - \cos\theta}$



why draw the curve thru the pole?

θ	0	$\pi/3$	$\pi/6$	$\pi/4$	$\pi/2$	π	$\frac{3}{4}\pi$	$\frac{2}{3}\pi$	$\frac{5}{6}\pi$	$\frac{15}{?}\pi$
$\theta°$	0°	60°	30°	45°	90°	180°	135°	120°	150°	[illegible]
$\cos\theta$	1	$\frac{1}{2}$	$\sqrt{3}/2$	$\sqrt{2}/2$	0	-1	$-\sqrt{2}/2$	$-\frac{1}{2}$	$-\sqrt{3}/2$	.9
r	4	2.67	3.53	3.14	2	1.33	1.47	1.60	1.55	3.8

Pg 15

1) $|2x+1| = 3$

$2x+1 = -3 \qquad 2x+1=3$

$2x = -4 \qquad 2x = 2$

$x = -2 \qquad x = 1$

$x = \{1, -2\}$

9) $\left|\frac{2x-3}{3x-2}\right| = 2$

$\frac{2x-3}{3x-2} = 2 \qquad \frac{2x-3}{3x-2} = -2$

$6x-4 = 2x-3 \qquad -6x+4 = 2x-3$

$4x = 1 \qquad 7 = 8x$

$x = \frac{1}{4} \qquad x = \frac{7}{8}$

$x = \{\frac{1}{4}, \frac{7}{8}\}$

11) $|x-2| < 1$

$0 = x-2$

$|0| < 1 \rightarrow 0 < 1 \cup -0 < 1$

$x-2 < 1 \cup 2-x < 1$

$x < 3 \cup -x < -1$

$x < 3 \cup x > 1$

$x = (1, 3)$

18) $\left|\frac{2x-5}{x-6}\right| < 3$

$-3 < \frac{2x-5}{x-6} < 3 \qquad -3 < \frac{2x-5}{x-6} < 3$

$x-6 > 0 \qquad x-6 < 0$

$-3x+18 < 2x-5 < 3x-18 \qquad -3x+18 > 2x-5 > 3x-18$

$-3x+18 < 2x-5 \qquad 2x-5 < 3x-18 \qquad -3x+18 > 2x-5 \qquad 2x-5 > 3x-18$

$23 < 5x \qquad 13 < x \qquad 23 > 5x \qquad 13 > x$

$\frac{23}{5} < x \qquad \frac{23}{5} > x$

$x > \frac{23}{5} \qquad x > 13 \qquad x < \frac{23}{5} \qquad x < 13$

$x > 13 \qquad x < \frac{23}{5}$

$(-\infty, \frac{23}{5}) \cup (13, \infty)$

10) $f(x) = \sqrt{(x-1)(x-3)}$

$(x-1)(x-3) \geq 0$

1 3

$(-\infty, 1] \cup [3, \infty)$

11) $f(x) = \sqrt{2-2x-x^2}$

$2-2x-x^2 \geq 0$

$x^2+2x-2 \leq 0$

$(x^2+2x+1)-3 \leq 0$

$(x+1)^2 \leq 3$

$x = \sqrt{3}-1 \qquad x = -\sqrt{3}-1$

$-\sqrt{3}-1 \qquad \sqrt{3}-1$

$[-\sqrt{3}-1, \sqrt{3}-1]$

14) $f(x) = x^3$

$f(x+h) = (x+h)^3$

$\frac{(x+h)^3 - x^3}{h} = x^3 +$

15) $f(x) = \frac{1}{x}$

$f(x+h) = \frac{1}{x+h}$

$\frac{\frac{1}{x+h} - \frac{1}{x}}{h}$

$\frac{[x-(x+h)]h}{x^2+xh} = \frac{-h^2}{x(x+h)}$

Pg 37

15) $x^2 + xy = 1 \qquad 2x - y = 2 \qquad -y = 2-2x \qquad y = 2x-2$

$x^2 + x(2x-2) = 1$

$x^2 + 2x^2 - 2x - 1 = 0$

$3x^2 - 2x - 1 = 0$

$(3x+1)(x-1) = 0$

$x = -\frac{1}{3} \qquad x = 1$

$y = -\frac{8}{3} \qquad y = 0$

9) $x^2 + y^2 + 4x + 6y - 21 = 0$ $\quad P(-4, 5)$

$x_0 x + y_0 y + 2(x_0 + x) + 3(y_0 + y) - 21 = 0$

$-4x + 5y + 2(x - 4) + 3(y + 5) - 21 = 0$

$-4x + 5y + 2x - 8 + 3y + 15 - 21 = 0$

$-2x + 8y - 14 = 0$

$-2x = 14 - 8y$

$2x = 8y - 14$

$x = 4y - 7$

$(4y - 7)^2 + y^2 + 4(4y - 7) + 6y - 21 = 0$

$16y^2 - 56y + 49 + y^2 + 16y - 28 + 6y - 21 = 0$

$17y^2 - 46y = 0 = y(17y - 46)$

$y = \{0, \frac{46}{17}\}$

$x = \{-7, \frac{65}{17}\}$

$(-4, 5)\ (-7, 0)$

$m = \frac{5}{3}$

$y = \frac{5}{3}(x + 7)$

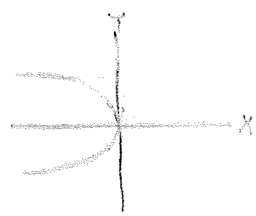
$3y = 5x + 35$

Pg 108

5) $5y^2 = -7x$

$y^2 = \frac{-7}{5}x$

$p = \frac{-7}{10}$



focus $= \{0, \frac{-7}{20}\}$

dir $\Rightarrow x = \frac{7}{20}$